Cusp Catastrophe Model

A Nonlinear Model for Health Outcomes in Nursing Research

Ding-Geng (Din) Chen ▾ Feng Lin ▾ Xinguang (Jim) Chen ▾ Wan Tang ▾ Harriet Kitzman

Background: Although health outcomes may have fundamentally nonlinear relationships with relevant behavioral, psychological, cognitively, or biological predictors, most analytical models assume a linear relationship. Furthermore, some health outcomes may have multimodal distributions, but most statistical models in common use assume a unimodal, normal distribution. Suitable nonlinear models should be developed to explain health outcomes.

Objective: The aim of this study is to provide an overview of a cusp catastrophe model for examining health outcomes and to present an example using grip strength as an indicator of a physical functioning outcome to illustrate how the technique may be used. Results using linear regression, nonlinear logistic model, and the cusp catastrophe model were compared.

Methods: Data from 935 participants from the Survey of Midlife Development in the United States (MIDUS) were analyzed. The outcome was grip strength; executive function and the inflammatory cytokine interleukin-6 were predictor variables.

Results: Grip strength was bimodally distributed. On the basis of fit and model selection criteria, the cusp model was superior to the linear model and the nonlinear logistic regression model. The cusp catastrophe model identified interleukin-6 as a significant asymmetry factor and executive function as a significant bifurcation factor.

Conclusion: The cusp catastrophe model is a useful alternative for explaining the nonlinear relationships commonly seen between health outcome and its predictors. Considerations for the use of cusp catastrophe model in nursing research are discussed and recommended.

Key Words: cusp catastrophe model • health outcomes • stochastic nonlinear model

The statistical model used to examine a health outcome in nursing research is typically based on a linear regression approach. However, the influence of environmental, behavioral, psychological, or biological factors on health outcomes are often complicated and nonlinear (Ray, 1998). Small and inconsequential changes in predictive factors may lead to abrupt changes in health outcomes. Under these conditions, the linear approach would seriously limit knowing the effects of factors hypothesized to be relevant to a health outcome. Other natural extensions of the linear regression to incorporate nonlinearity are nonparametric regression methods, such as the kernel regression or regression/smoothing splines in low-dimensional scenarios. For high-dimensional data, techniques such as the additive models, multivariate adaptive regression splines, random forests, neural networks, and support vector machine, etc., which have been discussed extensively in Faraway (2006), are available. However, these nonparametric regressions do not have the mechanisms to identify and incorporate "cusp jumps," which are the fundamental advantages of the cusp catastrophe models.

The cusp catastrophe model is capable of handling complex linear and nonlinear relationships simultaneously using a high-order probability density function that has the advantage of being able to incorporate sudden behavioral jumps (Zeeman, 1976). Historically, the cusp catastrophe model has been applied to prediction of health behaviors or system quality and safety, such as attitudes and social behavior (Flay, 1978), therapy and program evaluation (Guastello, 1982), accident processes (Guastello, 1989), anxiety and performance (Hardy & Parfitt, 1991), cognitive development (van der Maas & Molenaar, 1992), selection of target behaviors (Bosch & Fuqua, 2001), adolescent alcohol use (Clair, 1998), changes in adolescent substance use (Mazanov & Byrne, 2006), complexity of drinking relapse (Witkiewitz & Marlatt, 2007), binge drinking among college students (Guastello, Aruka, Doyle, & Smerz,, 2008), early

Ding-Geng (Din) Chen, PhD, is Professor, School of Nursing and Department of Biostatistics and Computational Biology, University of Rochester Medical Center, New York, and Tianjin International Joint Academy of Biotechnology and Medicine, China.

Feng Lin, PhD, RN, is Assistant Professor, School of Nursing and Department of Psychiatry, School of Medicine and Dentistry, University of Rochester Medical Center, New York.

Xinguang (Jim) Chen, PhD, is Professor, Department of Epidemiology, College of Public Health and Health Professions and College of Medicine, University of Florida, Gainesville.

Wan Tang, PhD, is Associate Professor, Department of Biostatistics and Computational Biology, University of Rochester Medical Center, New York.

Harriet Kitzman, PhD, RN, FAAN, is Professor, School of Nursing, University of Rochester Medical Center, New York.

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sexual initiation among young adolescents (Chen et al., 2010), nursing turnover (Wagner, 2010), and HIV prevention (Chen, Stanton, Chen, & Li, 2013). The cusp catastrophe model, though, has seldom been applied to the understanding of health outcomes, such as the incidence of a disease or changes in a health condition where the nature can be extremely complicated and dynamic. The goal of this article is to provide an overview of the cusp catastrophe model, focusing on its application in the examination of health outcomes. Such a method can assist nurse researchers in taking the next steps in understanding the multifaceted nonlinear impact of different predictors on health outcomes in a new way. Findings based on the cusp catastrophe model may guide evaluation of outcomes from interventions more effectively than other methods.

Overview of the Cusp Catastrophe Model

Popularized in the 1970s by Thom (1975), catastrophe theory was originally proposed to explain complicated sets of behaviors that include both continuous changes and sudden discontinuous or catastrophical changes. Theoretically, five elements called catastrophe flags define the presence of catastrophe (Gilmore, 1993): (a) bimodality, where two distinctly different modes exist in the distribution of the outcome; (b) sudden jump, where the outcome changes abruptly between the modes even with slight changes in the predictors; (c) inaccessibility, where an outcome value in the area between the modes is unlikely; (d) hysteresis, where change from one mode to the other cannot be determined by the same values for control factors; and (e) divergence, where a slight change in the control factors can lead to substantial change in the outcome and deviation from the linear model. In summary, a cusp catastrophe model would be particularly appropriate if am outcome measure has the properties of a bimodal distribution (*bimodality*) with spurts (*sudden jump*) along with a middle inaccessible region between two modes (*inaccessibility*) with delay between transitions (*hysteresis*) and deviation from a linear relationship between the response outcome measure and the predictors (*divergence*). Further definition and explanations are summarized in Table 1.

Although cusp catastrophe models have been well established theoretically and extensively applied to physical sciences, cusp catastrophe models were criticized in the early 1970s in applications in social and behavior sciences partially because mathematics were misused, models were based on unreasonable assumptions, and predictions were thought to be vague or impossible to test experimentally (e.g., Sussmann & Zahler, 1978, p. 118), charges that were later reconsidered by Rosser (2007), who argued for utility of the cusp modeling approach for problems with dynamic discontinuities in outcomes. It is of interest in nursing in part because it is associated with theories proposed by Rogers (1971).

The deterministic cusp catastrophe model is specified using three components: two control factors (i.e., x and y) and one outcome variable (i.e., z). This model is defined by a differential equations-based dynamic system:

$$\frac{dz}{dt} = -\frac{dV(z;x,y)}{dz} \tag{1}$$

where the potential function is

$$V(z;x,y) = \tfrac{1}{4}z^4 - \tfrac{1}{2}z^2 y - zx.$$

For the function V, the argument x is called asymmetry or normal control factor where the outcome z changes asymmetrically from one mode to the other eventually as x increases, y is called bifurcation or splitting control factor, which causes the outcome surface to split and bifurcate from smooth changes to sudden jumps as y increases. Both x and y are linked to determine the outcome variable z in a three-dimensional outcome response surface. When the right side of Equation 1 moves toward 0, the outcome z will not change with time. Such status is called equilibrium; this assumption is needed to interpret cusp models based on cross-sectional data. In general, the behavior of the outcome z, that is, how it changes with time t, is in general complicated, but each subject will move toward an equilibrium status. Figure 1 graphically depicts the equilibrium plane, which reflects the response surface of the outcome measure (z) at various combinations of asymmetry control factor (x) and bifurcation control factor (y).

Cusp catastrophe models can be assessed qualitatively and quantitatively. The qualitative approach (Gilmore, 1993) focuses on identification of catastrophe flags. This study focuses on use of the quantitative approach, called "stochastic cusp catastrophe model." The quantitative approach extends the deterministic cusp model in Equation 1 by adding a probabilistic/stochastic Wiener process to incorporate the measurement errors of the outcome measurement. The response surface of the cusp catastrophe model can be modeled as a probability density function where the bimodes of the outcome correspond to two states of health outcome. Statistically, the deterministic cusp model in Equation 1 is cast into a stochastic differential equation (Cobb, 1981; Cobb & Ragade, 1978; Cobb & Watson, 1980; Cobb & Zacks, 1985) as follows:

$$dz = \frac{\partial V(z;x,y)}{\partial z}dt + dW(t) \tag{2}$$

where $dW(t)$ is a white noise Wiener process with variance σ^2, which is in fact a special case of the general stochastic dynamical system model with constant diffusion function defined with $dW(t)$ as a white noise Wiener process with variance σ^2. This model is still mathematically complex, and the analytical solution to this stochastic differential equation in Equation 2 cannot be obtained feasibly. Therefore, its computational implementation and real-life application to health outcomes research are limited. However, as time (t) passes, the probability density function of the corresponding limiting stationary stochastic

TABLE 1. Definitions of Key Terms

Term	Definition
Cusp catastrophe model (cusp model)	Catastrophe theory is a branch of bifurcation theory in the study of dynamical systems to study phenomena characterized by sudden shifts in behavior from small changes in circumstances where cusp catastrophe model is one of the catastrophe models in this area as discussed in Zeeman (1976).
Asymmetry control factor	In cusp model, there are two control factors to control the outcome response surface where the asymmetry control factor is used to control the outcome changes asymmetrically from one mode to the other mode eventually as it increases as seen in Figure 1.
Bimodality of outcomes	Human health or behavior outcomes, such as grip strength, nursing turnover (Wagner, 2010), adolescent alcohol use (Clair, 1998), and adolescent sex behavior (Chen et al. 2013), are often bimodally distributed.
Bifurcation control factor	Similarly to the "Asymmetry control factor," the bifurcation control factor controls the outcome surface to split and bifurcate from smooth changes to sudden jumps as it increases as seen in Figure 1.
Equilibrium	Equilibrium is a state from the dynamic system in Equation 1 where the outcome (z) does not change with time.
Potential function	The potential function is a general technical term used in dynamic systems models as a function to relate the outcome to any other control factors.
Inaccessibility	A rare if not impossible intermediate state between two opposite behavior modes.
Sudden jump	Outcomes change suddenly between the two modes with slight changes in the bifurcation and asymmetry control factors close to the cusp region as depicted in Figure 1.
Hysteresis	The change of outcomes from one mode to the other is impossible to be determined by the same value of bifurcation and asymmetry control factors where the sudden jumps do not always occur at the same value of these control factors.
Divergence	A slight change in the bifurcation control factor can lead to substantial change in the outcome and two possible paths are available with increasing values of the bifurcation control factor.
Wiener process	In health outcome research, a real-time process where changes in the outcome over an increment of time have a known normal distribution
Deviant to linear model	Outcomes change paths from smooth linear mode to nonlinear with sudden jumps as the bifurcation factor changes.

processes (i.e., equilibrium) is easier to obtain. More precisely, the probability density function of the outcome measure (z) (Hartelman, 1997; Honerkamp, 1994) can be expressed as follows:

$$f(z) = \frac{\psi}{\sigma^2} exp\left[\frac{x(z-\lambda) + \frac{1}{2}y(z-\lambda)^2 - \frac{1}{4}(z-\lambda)^4}{\sigma^2}\right] \quad (3)$$

where the parameter ψ is a normalizing constant and λ is used to determine the origin of z. With this formulation of probability density function, the regression predictors can be incorporated as linear combinations to replace the canonical asymmetry factor (i.e., x) and bifurcation factor (i.e., y), as shown in Equations 6 and 7. Note that, as a distribution for a limiting stationary stochastic process, it is independent from time t; thus, it can be used to model a cross-sectional relationship with the advantage to detect and quantify its potential cusp nature comprising both sudden and continuous states. With this probability density function, the well-known statistical theory of maximum likelihood can be readily employed for model parameter estimation and statistical inference. The *R* package "cusp" has been developed to implement the stochastic cusp catastrophe model (Grasman, van der Mass, & Wagenmakers, 2009).

Potential usefulness of the cusp model can be illustrated by fitting linear regression, nonlinear regression using a logistic function, and cusp models and then comparing fit and interpretability of the parameters. Comparatively smaller values of the negative log-likelihood, associated likelihood ratio, chi-square tests, Akaike Information Criterion (Akaike, 1974), and Bayesian Information Criterion (Gelfand & Dey, 1994) indicate better fit. Higher pseudo-R^2 values demonstrate higher explained variance in the outcome and are interpreted as R^2 in a linear regression. In a cusp model, at least 10% of the control factor data pairs (x, y) should lie within the bifurcation cusp region (Cobb, 1998; Hartelman, 1997). A more stringent alternative for this 10% guideline was proposed by Hartelman (1997); Hartelman, van der Maas, and Molenaar (1998); and van der Maas, Kolstein, and van der Pligt (2003) to be a nonlinear least squares regression with the logistic curve (thereafter referred to as "nonlinear logistic model") as follows:

$$z = \frac{1}{1 + e^{-x/y^2}} + \varepsilon \quad (4)$$

where (x,y,z) are defined in Equations 6–8 in the following section for sample size N. This nonlinear logistic model has the

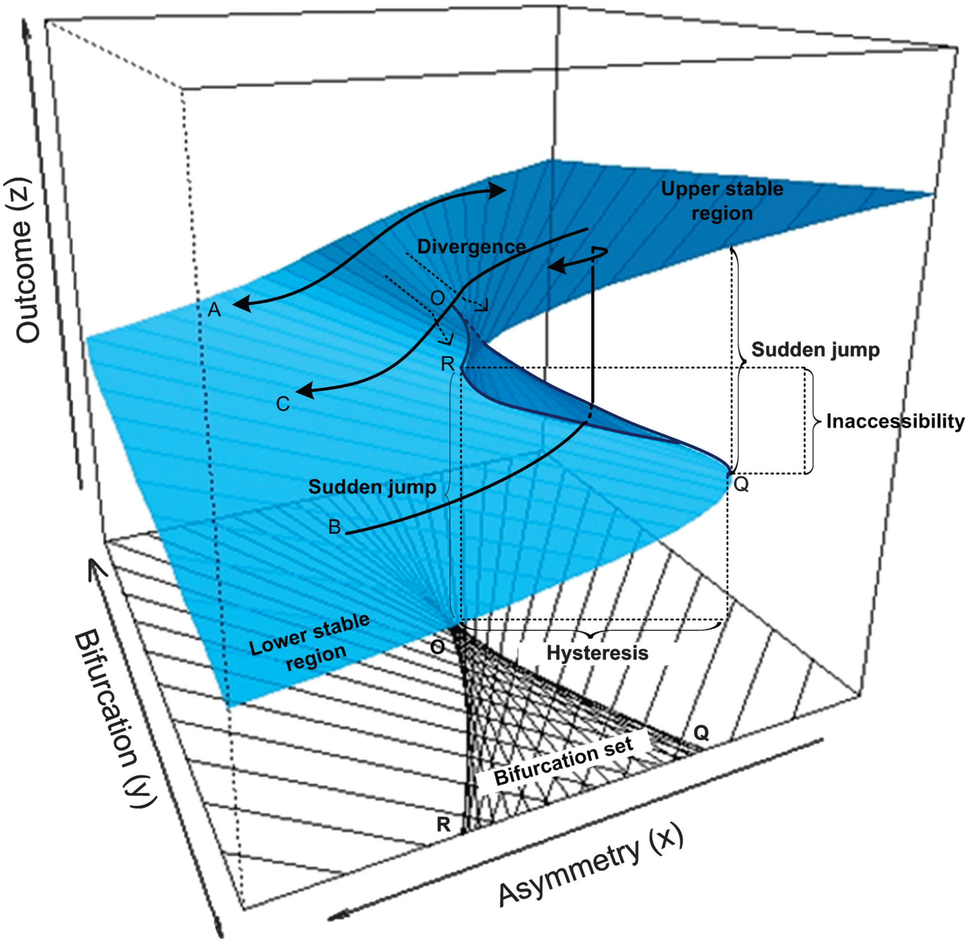

FIGURE 1 Cusp catastrophe model for health outcome measures (z) in the equilibrium plane with asymmetry control variable (x) and bifurcation control variable (y). The dynamic changes in z have two stable regions (attractors), which are the lower area in the front left (lower stable region) and the upper areas in the front right (upper stable region). Beyond these stable regions, z becomes sensitive to changes in x and y. This unstable region can be projected to the control plane (x, y) as the cusp region. The cusp region is characterized by line O–Q (the ascending threshold) and line O–R (the descending threshold) of the equilibrium surface. In this region, z becomes highly unstable with regard to changes in x and y, jumping between the two stable regions when (x, y) approaches the two threshold lines O–Q and O–R. In this figure, Paths A, B, and C depict three typical but different pathways of change in the health outcome measure (z). Path A shows that in situations where $y < 0$, there is a smooth relation between z and x; Path B shows that in situations where $y > 0$, if x increases to reach and pass the ascending threshold link O–Q, z will jump suddenly from the low stable region to the upper stable region of the equilibrium plane; Path C shows a sudden drop in z as x declines to reach and pass the descending threshold line O–R.

advantage to model the steep changes in the outcome variable to mimic the "sudden" transition in the cusp.

CUSP CATASTROPHE MODEL IN HEALTH AND NURSING RESEARCH

As shown by publications cited earlier in the article, many health outcomes may satisfy the five criteria (catastrophe flags) proposed in catastrophe theory. In clinical practice, for example, many physical and mental health conditions (e.g., seizures, cardiac arrest, stroke, depression, or bipolar disorders) have two modes: normal versus abnormal (*bimodality*) and a low probability of "between area" beyond the two modes (*inaccessibility*). These health conditions also seem to suddenly jump from one end to the other (*sudden jump*, as when a condition is diagnosed based on severity). Often, relatively small changes in environmental factors, biological and psychological status, or behaviors can induce or trigger sudden and dramatic changes in the status of health conditions (*divergence*), and the timing and direction of these predictors would determine the incidence or severity of health outcome (*hysteresis*). For example, a small amount of air entering the bloodstream may sometimes precipitate stroke or cardiac arrest; yet, how often such adverse events occur depends on certain circumstances, such as the speed of and location for injecting the air (Schottke, 2010).

Statistically, compared to a linear regression and nonlinear logistic model in Equation 4, the advantages of applying a cusp catastrophe model in studying health outcomes are as follows:

1. The cusp model allows the forward and backward progression following different paths in health outcomes to be modeled simultaneously (see Paths B and C in Figure 1), whereas a linear model only permits one type of relationship to be modeled;
2. The cusp model covers both a discrete component (normal vs. abnormal) and a continuous component (the degree of severity) of the health outcomes where the linear model is a special case of this continuous component. The continuous part manifests the linear and gradual process (Path A), and the discrete part characterizes the sudden and nonlinear process (Paths B and C). A linear model can only capture the continuous part.
3. The cusp model consists of two stable regions and two thresholds where sudden changes occur (upper and lower regions in Figure 1). A linear model does not have these features.

EXAMPLE

Relationships between executive function (EF), interleukin-6 (IL-6), and grip strength are used to demonstrate application of the cusp catastrophe model to health outcomes. Grip strength in adults is an indicator of physical functioning, especially musculature and weakness, and is a core component of frailty (Roberts et al., 2011). Importantly, grip strength is a critical predictor of hospitalization, independent living, and mortality (Landi et al., 2013). Thus, there is growing interest in understanding determinants of grip strength in adults. The following characteristics of grip strength are consistent with the criteria for catastrophe flags:

1. *Bimodality* where there are two states—"sarcopenia/impaired" versus "normal" or a continuous form of strength (Cruz-Jentoft et al., 2010).
2. There is a low probability of accessing the intermediate state of weakness between the "sarcopenia/impaired" and "normal" mode (*inaccessibility*).
3. It is possible to reach the two states of grip strength by increasing or decreasing certain physiological factors (*divergence*).
4. Slight changes in individuals' physiological factors may cause a *sudden jump* between the two grip strength states (Cruz-Jentoft et al., 2010).
5. The variability in physiological factors usually leads to different grip strength states (*hysteresis*).

Inflammation is known to have degrading effects on bone and muscle mass. Such effects are thought to contribute to muscle weakness by accelerating protein loss and contractile dysfunction (Beyer et al., 2012). However, findings on the relationship between proinflammatory cytokines, such as IL-6, and grip strength have been inconsistent across studies (Payette et al., 2003; Schaap, Pluijm, Deeg, & Visser, 2006). Muscle stength depends in part on brain control, so the cognitive operation of EF may interact with levels of inflammatory processes (indicated by IL-6) to explain individual differences in grip strength state (MacDonald, DeCarlo, & Dixon, 2011). Although the interaction has been traditionally examined using multiple linear regression analysis, the characteristics of grip strength described above suggest that this condition could be analyzed with the cusp catastrophe model to determine the impact of IL-6 and EF. Thus, in the following cross-sectional design secondary data analysis, three models were compared: (a) multiple linear regression, (b) nonlinear logistic model, and (c) the cusp catastrophe model.

Data for this example were obtained from the second wave of data on Survey of Midlife Development in the United States (MIDUS II), an ongoing nationally representative longitudinal survey data set. MIDUS II is the 10-year follow-up study of MIDUS I (a longitudinal study of physical and psychological health of adults in the United States). A total of 4,963 participants from MIDUS I participated in demographic and psychobehavioral assessments (i.e., MIDUS II Project 1), demonstrating a 75% retention rate adjusted for mortality. In addition, four substudies were included in MIDUS II. Project 2 involved the completion of daily dairies to track daily stressors; Project 3 involved the assessment of cognitive functioning, Project 4 involved the collection of biomarkers and physical assessments, and Project 5 involved the completion of brain functioning assessments. Institutional review board approval was obtained for each study project at each study site, and informed written consent was obtained from all participants (Dienberg Love, Seeman, Weinstein, & Ryff, 2010). Data used in this example were from MIDUS II Projects 3 and 4. MIDUS data set is under the category of Interuniversity Consortium for Political and Social Research. Therefore, the present secondary data analysis study did not require prior institutional review board approval. There were 935 participants who participated in both the cognition and biomarker projects and had complete data for inflammatory cytokines, EF, and grip strength.

As described in Dienberg et al. (2010), five tests were used to measure EF: (a) working memory span (digits backwards), (b) verbal fluency (category fluency), (c) inductive reasoning (number series), (d) processing speed (backward counting from the Brief Tests of Adult Cognition by Telephone), and (e) attention switching and inhibitory control from the Stop and Go Switch Task. An average of *z* scores for all tests was used as a composite score for EF in the data analysis (Lachman, Agrigoroaei, Murphy, & Tun, 2010). IL-6 was measured using Quantikine high-sensitivity enzyme-linked immunosorbent assay kits (R&D Systems, Minneapolis, MN). The laboratory intra-assay coefficient of variance was 13% for IL-6. Grip strength was assessed using a handheld dynamometer. The average of three trials in the dominant hand was used.

The average age of the sample was 58.15 (SD = 11.62, range = 35–86). Around half were female (54.4%), and two thirds graduated from high school (75.7%); 34.4% were taking antihypertensive, 12.4% were taking corticosteroid, and 14.8% were taking antidepressant; 11% of the participants were active smokers, and 40.0% had alcohol intake at least once a week.

To implement the cusp model, all covariates, including age, gender, education, antidepressant, corticosteroids, antihypertension, smoking, and alcohol intake, were adjusted to grip strength using a linear regression. The adjusted grip strength (predictive value) was then used in the cusp model as follows. The multiple linear regression was simply defined as

$$\text{Grip Strength} = \beta_0+\beta_1 \text{ IL-6}+\beta_2 \text{ EF}+\beta_3 \text{ (IL-6}\times\text{EF)}. \quad (5)$$

To implement the cusp catastrophe model in Equation 2, both the asymmetry control factor x and the bifurcation factor y as a linear combination of the predictor variables of EF and IL-6 were initially defined as

$$x = a_0 + a_1 \text{ EF} + a_2 \text{ IL-6} \quad (6)$$

$$y = b_0 + b_1 \text{ EF} + b_2 \text{ IL-6} \quad (7)$$

where the intercept coefficients a_0 and b_0 link to the mean effect of all predictor variables on the dependent variable of grip strength (z) and other coefficients assess the independent effects of these variables on grip strength (z). The dependent variable of health measure (z) is also defined as a linear equation:

$$z = w_0 + w_1 \text{ Grip Strength} \quad (8)$$

where w_0 and w_1 represent the mean level and change in health measure along with the two latent control variables x and y defined above.

Table 2 shows the comparison between multiple linear regression in Equation 5, the nonlinear logistic model in Equation 4, and the cusp catastrophe model in Equation 2. The linear regression is significant ($p < .0001$); if only the multiple linear regression in Equation 5 had been estimated, it would lead to the conclusion of a significant overall regression model. By fitting the linear regression model, nonlinear logistic model, and cusp models and comparing them, it was found that the asymmetry control factor (x) was driven by IL-6 (EF had no effect on asymmetry, the coefficient was nonsignificant) and the bifurcation control factor (y) was driven by EF (the coefficient for IL-6 was nonsignificant) as seen in Table 2.

First of all, the data fit the cusp model well (R^2 = .79). IL-6 as the only significant variable for the asymmetry control factor was negatively associated with grip strength (a_1 = −0.1349, $p < .01$). EF as the only significant variable for the bifurcation control factor was negatively associated with greater grip strength (b_1 = −0.1599, $p < .05$).

Comparatively, the cusp catastrophe model was superior to the linear model and the nonlinear logistic model; values of the Akaike Information Criterion and Bayesian Information Criterion model selection criteria were lowest for the cusp model (Table 2). In addition, R^2 increased from .05 in the linear model in Equation 5 and .06 in the nonlinear

TABLE 2. Parameter Estimates and Model Comparison for Prediction of Grip Strength

	Parameter			Model Fit and Selection Criteria			
Model	**Name**	**Estimate**	***p***	***R*²**	**–LL**	**AIC**	**BIC**
Logistic[a]	a_0: intercept coefficient for IL-6	0.528	.85	.06	3,408.11	6,826.22	2,332.73
	b_0: slope coefficient for IL-6	1.000	<.001				
	a_1: intercept coefficient for EF	–0.057	<.001				
	b_1: slope coefficient for EF	–0.115	.003				
Linear[b]	b_0: intercept coefficient	0.001	.99	.05	3,408.94	6,825.93	6,845.25
	b_1: slope coefficient for IL-6	1.048	.02				
	b_2: slope coefficient for EF	2.313	<.001				
	b_3: coefficient for IL-6 × EF interaction	–0.162	.73				
Cusp[c,d]	a_0: intercept coefficient for IL-6	0.0004	.99	.79	1,145.84	2,303.68	2,332.73
	b_0: intercept coefficient for EF	1.770	<.001				
	w_0: intercept coefficient for grip strength	–4.8348	<.001				
	a_1: slope coefficient for IL-6	–0.1349	<.001				
	b_1 : slope coefficient for EF	–0.1599	.01				
	w_1: slope coefficient for grip strength	0.1271	<.001				

Note. AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion; EF = executive function; IL = interleukin; LL = log-likelihood. [a]Equation 4. [b]Equation 5. [c]Equation 2. [d]IL-6 cytokine value was used as the asymmetry control variable x, and EF was used as the bifurcation variable y.

logistic model in equation 4 to .79 in the cusp catastrophe model in Equation 2.

The distribution of the grip strength scores and model residuals were also examined (see Figure 2). The distribution of grip strength was not normal, and there were two distinct modes at grip strength values of about 28 and 48 (Figure 2A). The residual plot from the linear regression (Figure 2B) revealed a nonnormal distribution ranging from −20 to 20, indicating that the linear regression assumption of normally distributed residuals was violated. If the results from the linear regression were interpreted, erroneous conclusions would be reported. Differently, the residual plot from the cusp model (Figure 2C) is normally distributed and ranges from −2 to 2.

In summary, the cusp catastrophe model showed better data-model fit as seen in Table 2 and especially captured the data heterogeneity as shown in Figure 2 in comparison with the classical linear and nonlinear logistic model. Results of the analysis supported the conclusion that the cusp catastrophe modeling method was superior to the traditional linear and nonlinear approaches in characterizing the nonlinear changes in grip strength, assuming a dynamic system at equilibrium. Plotting the data distribution of grip strength revealed two modes: "strength"/"normal" versus "weak"/"sarcopenia." In general, there was a negative association between IL-6 and grip strength, suggesting that the grip strength was stronger for patients with less severe inflammation (lower IL-6) and weaker for patients with more severe inflammation (higher IL-6). However, this relationship took two modes, depending on the levels of EF. When EF is at its higher levels, the negative association between IL-6 and grip was gradual and continuous (Path A in Figure 1). When EF was low, the relationship between IL-6 and grip strength became complicated: (a) EF may interact with low degrees of IL-6 to maintain strong grip strength, shown in the upper stable region in Figure 1; (b) EF may become dysfunctional with high grades of IL-6, resulting in comprised grip strength as shown in the lower stable region in Figure 1; and (c) as IL-6 level varied across the region of the bifurcation set, sudden deterioration or improvement of grip strength was inducible in response to even subtle changes in IL-6 (Paths B and C in Figure 1).

DISCUSSION

In this research, the cusp catastrophe model was introduced with an example that characterizes the potential dynamic process of psychological (i.e., EF) and biological (e.g., IL-6) factors in affecting the functional health outcome of grip strength. The goodness-of-fit model and residuals both supported application of this cusp catastrophe model when compared to a linear regression model and nonlinear logistic regression model. As evidenced in the example, the cusp catastrophe model may be superior to linear and nonlinear logistic regression models for quantifying discontinuous and bimodal health outcomes in nursing research. Preliminary assessment of data for detection of cusp

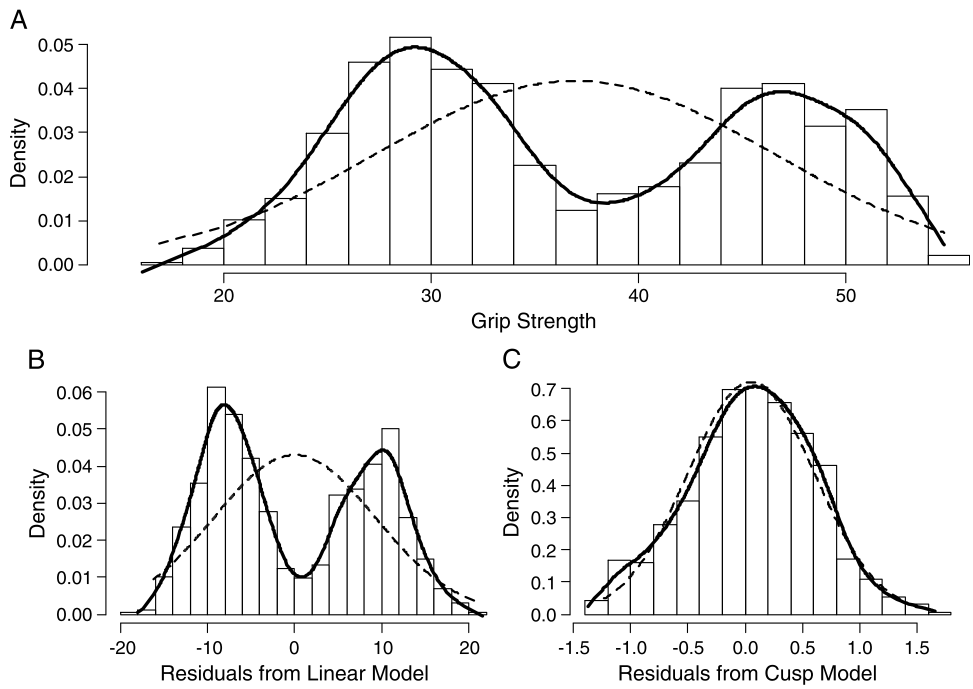

FIGURE 2 Distribution plots: (A) grip strength, (B) residuals from linear model, and (C) residuals from cusp model. The dashed lines indicate the normal distribution, and the solid lines are from the nonparametric spline-smooth based on the data.

catastrophes using Gilmore's (1993) five essential elements of bimodality, sudden jump, inaccessibility, hysteresis, and divergence, called catastrophe flag detection should be undertaken before fitting cusp models. When the cusp model seems plausible, parameters of a stochastic cusp catastrophe model can be estimated and used to identify asymmetry and bifurcation control factors. Compared to alternatives such as linear and nonlinear logistic models, the cusp model strengthens the overall approach, linking the health outcome to its predictors.

Applying a dynamic analysis using catastrophe theory may increase the ability to select and deliver interventions to address health outcomes by targeting specific combinations of risk factors. Patient-centered care may be incorporated more effectively into evidence-based practice. Adults who experience adverse health conditions, especially the older adults, may lack energy to manage multiple tasks/treatments simultaneously. It is important to identify the targets with highest priority, and to deliver the most effective treatment in a timely manner. To reach all these goals requires a dedicated plan to tailor the evidence-based practice to the characteristics of the individual.

Taking the grip strength case as an example, anti-inflammatory therapy and physical exercise (e.g., resistance training) are common evidence-based treatments for improving muscle strength (Beyer et al., 2011; Geirsdottir et al., 2012). Nevertheless, to maximize the treatment effect, individual differences, which is patient centeredness, should be taken into consideration. In this case, the patient centeredness indicator is cognitive function, particularly EF. For those individuals with relatively high levels of EF, traditional treatments (e.g., anti-inflammation medication, physical exercises) that potentially mitigate inflammatory activation should be the priority for maintaining muscle strength. Whereas, for individuals with compromised EF, anti-inflammatory treatment and improvement of EF are equally critical to achieve sudden improvement in grip strength. Improvement of EF is possibly achieved by using cognitive modification strategies (e.g., physically, mentally, socially active lifestyle, and healthy diet; Middleton & Yaffe, 2010). Given the possibility of a dramatic deterioration of muscle strength from inflammatory activation for those with low EF, it is also important to recognize that preventing inflammatory conditions and continuously monitoring relevant risk factors (e.g., blood sugar, weight; Kantor, Lampe, Kratz, & White, 2013) should be implemented on a regular basis—especially for those at risk for inflammatory diseases (e.g., older age, vascular risk; Puntmann, Taylor, & Mayr, 2011).

There are some limitations to this study. Despite the strength of the modeling approach used in this study to quantify the cusp dynamic process, it is a cross-sectional model because of the mathematical challenge to obtain analytical solution to the time-dependent (i.e., longitudinal) stochastic differential equation in Equation 2. As a trade-off to this theoretical challenge, a time-independent special case was adopted, which is now widely used in cusp catastrophe modeling as discussed in Grasman et al. (2009). Although this time-independent cusp catastrophe model is suitable for cross-sectional data from health outcomes research, it cannot be used to capture the within-individual changes over time. Statistical methods for longitudinal cusp catastrophe modeling are not available at the point when this study was completed. Likewise, the lack of methodologies to estimate the threshold lines and cusp regions for stochastic cusp modeling prevented us from quantifying these two model parameters. Despite these limitations, cusp catastrophe modeling is a useful technique to model the nonlinear relationship between the health outcome and its associated behavioral, psychological, cognitive, or biological predictors, capturing continuous and discontinuous changes in the health outcome. Nursing researchers may choose to utilize the cusp catastrophe model to examine the complex nonlinear process that is related to health outcomes over traditional linear techniques. In further research, the cusp catastrophe model will be extended to examine longitudinal data by incorporating the temporal correlation and develop the statistical power analysis strategies for this model as well as develop a statistical framework to determine the thresholds for cusp regions.

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Corresponding author: Ding-Geng (Din) Chen, PhD, School of Nursing and Department of Biostatistics and Computational Biology, University of Rochester Medical Center, 601 Elmwood Ave, Box SON, Rochester, NY 14642 (e-mail: DrDG.Chen@gmail.com).

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